James' Compactness Theorem with Mackey's Constraints

J. Orihuela

We shall present the following result:

Theorem 1 *Let A be a closed, convex, bounded and not weakly compact subset of a Banach space E. Let us fix an absolutely convex and weakly compact subset W of E^{**}, a functional $z_0^* \in E^*$ and $\epsilon > 0$. Then there is a linear form $x_0^* \in B_{p_W}(z_0^*, \epsilon)$, i.e.*

$$|x_0^*(w) - z_0^*(w)| < \epsilon$$

for all $w \in W$, which does not attain its supremum on A.

This result answers a question posed by M. Jimenez and J.P Moreno [1] for the weak topology, i.e. when W reduces to a finite subset of E^{**}.

References

[1] M. Jimenez, J.P. Moreno, A note on norm attaining functionals. Proc. Am. Math. Soc. (1998) 126, 1989–1997.